



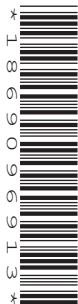
Oxford Cambridge and RSA

Thursday 22 May 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Time allowed: 2 hours 40 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **144**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Section A (33 marks)

- 1 The complex number z satisfies the equation $z + 2iz^* + 1 - 4i = 0$.

You are given that $z = x + iy$, where x and y are real numbers.

Determine the values of x and y . [4]

- 2 In this question you must show detailed reasoning.

Find the acute angle between the planes $2x - y + 2z = 5$ and $x + 2y + z = 8$. [4]

- 3 Using standard summation formulae, show that, for integers $n \geq 1$,

$$1 \times 3 + 2 \times 4 + \dots + n \times (n+2) = \frac{1}{6}n(n+1)(an+b),$$

where a and b are integers to be determined. [5]

- 4 (a) You are given that $\mathbf{M}$ and $\mathbf{N}$ are non-singular 2×2 matrices.

Write down the product rule for the inverse matrices of $\mathbf{M}$, $\mathbf{N}$ and $\mathbf{MN}$. [1]

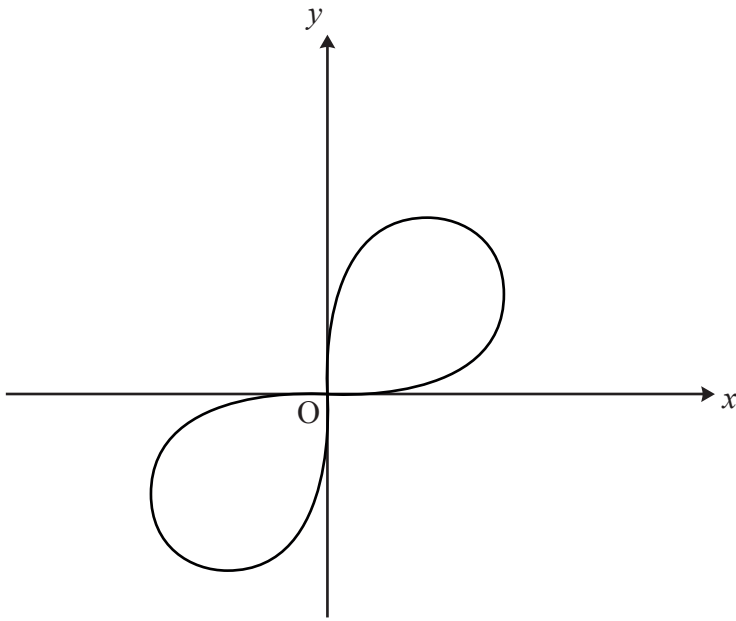
- (b) Verify this rule for the matrices $\mathbf{M}$ and $\mathbf{N}$, where

$$\mathbf{M} = \begin{pmatrix} a & 1 \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} 0 & -1 \\ 1 & b \end{pmatrix} \text{ and } a \text{ and } b \text{ are non-zero constants.} [6]$$

- 5 The cubic equation $2x^3 - 3x + 4 = 0$ has roots α , β and γ .

Determine a cubic equation with integer coefficients whose roots are $\frac{1}{2}(\alpha + 1)$, $\frac{1}{2}(\beta + 1)$ and $\frac{1}{2}(\gamma + 1)$. [4]

- 6 The figure below shows the curve with cartesian equation $(x^2 + y^2)^2 = xy$.



- (a) Show that the polar equation of the curve is $r^2 = a \sin b\theta$, where a and b are positive constants to be determined. [3]
- (b) Determine the exact maximum value of r . [2]
- (c) Determine the area enclosed by **one** of the loops. [4]

Section B (111 marks)

7 In this question you must show detailed reasoning.

By first expressing $\frac{1}{x^2-4}$ in partial fractions, show that $\int_3^\infty \frac{1}{x^2-4} dx = \frac{1}{m} \ln n$, where m and n are integers to be determined. [8]

8 The function $f(x)$ is defined as $f(x) = \ln(1+x)$, for $x > -1$.

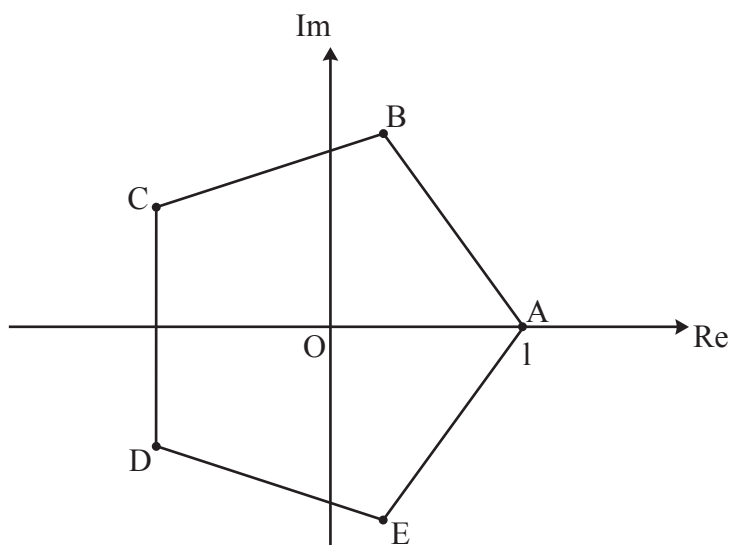
(a) Prove by mathematical induction that the n th derivative of $f(x)$, $f^{(n)}(x)$, for all $n \geq 1$, is given

$$\text{by } f^{(n)}(x) = \frac{(-1)^{n+1}(n-1)!}{(1+x)^n}. \quad [4]$$

(b) Hence prove that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^{n+1}x^n}{n} + \dots$ for $-1 < x \leq 1$.

[You are not required to show this series for $\ln(1+x)$ converges for $-1 < x \leq 1$.] [3]

- 9 The figure below shows an Argand diagram with a **regular** pentagon ABCDE. The point A represents the real number 1. The point B represents the complex number w .



- (a) (i) Write down, in terms of w , the complex numbers represented by the points C, D and E. [1]
- (ii) Write down an equation whose roots are the complex numbers represented by the points A, B, C, D and E. [1]
- (iii) Show that the sum of these roots is zero. [2]
- (b) (i) Find w . Give your answer in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $\theta = k\pi$, where k is a positive constant to be found. [1]
- (ii) By considering the line segment AB, show that the length of each side of the pentagon is $2 \sin \frac{\pi}{5}$. [5]

10 In this question you must show detailed reasoning.

Evaluate $\int_0^{\frac{1}{2}} \frac{2}{x^2 - x + 1} dx$. Give your answer in exact form. [4]

- 11 The lines l_1 and l_2 have equations

$$l_1: \frac{x-3}{a} = \frac{y+1}{b} = \frac{z-2}{1} \quad l_2: \mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + c\mathbf{j} + 2\mathbf{k})$$

where a , b and c are constants.

- (a) In the case where $c = 0$ and l_1 and l_2 intersect at right angles, determine the coordinates of the point of intersection of the two lines. [6]
- (b) Now consider the case where $c = 4$ and lines l_1 and l_2 are parallel.
- (i) Find the values of a and b . [3]
- (ii) Determine the distance between the two lines. [5]

- 12 In this question you must show detailed reasoning.

The roots of the equation $z^4 - z^3 + cz^2 + dz + 18 = 0$ are α , $\frac{2}{\alpha}$, β and $-\beta$.

Determine, in any order, the exact values of the following.

- The **four** roots of the equation
- The value of c
- The value of d [8]

- 13 The gradient of a curve $y = f(x)$ satisfies the differential equation $x \frac{dy}{dx} - 2y = 2 + x^2$.

- (a) Show that the integrating factor for this differential equation is x^{-2} . [3]
- (b) You are given that the curve passes through the point $(1, 0)$.

By solving this differential equation, determine the exact x -coordinate of the stationary point on the curve $y = f(x)$. [8]

- 14 (a) By using the definition of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x} , show that $\cosh^2 x + \sinh^2 x \equiv \cosh 2x$. [2]

- (b) The transformation T of the plane has associated matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{pmatrix}$ and $x > 0$.

Show that T transforms the unit square with coordinates $(0, 0)$, $(1, 0)$, $(0, 1)$ and $(1, 1)$ to a rhombus of unit area. [6]

- (c) You are given that the length of each side of the rhombus is 2 units.

Determine the exact value of x . Give your answer in logarithmic form. [2]

- 15 (a) Show that $(3 - e^{4i\theta})(3 - e^{-4i\theta}) = a + b \cos 4\theta$, where a and b are integers to be determined. [2]

The infinite series C and S are defined as follows.

$$C = \cos \theta + \frac{1}{3} \cos 5\theta + \frac{1}{9} \cos 9\theta + \frac{1}{27} \cos 13\theta + \dots$$

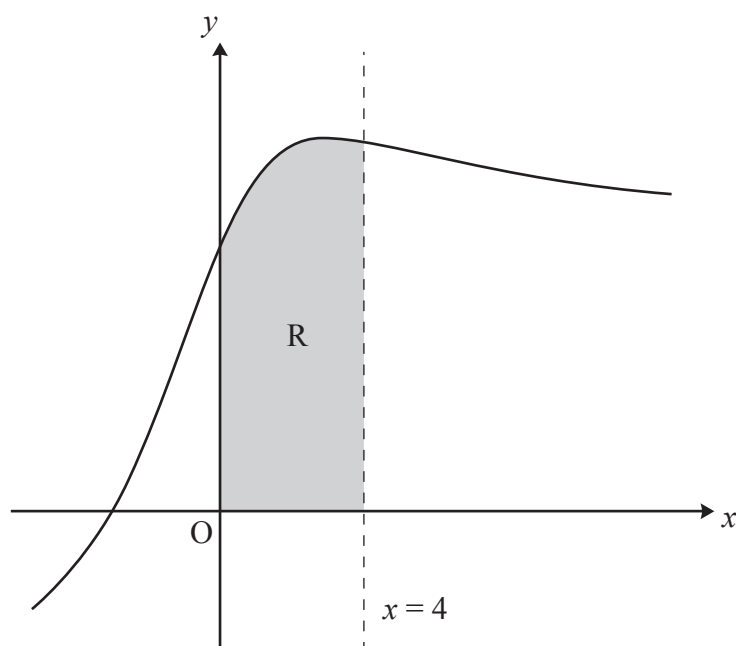
$$S = \sin \theta + \frac{1}{3} \sin 5\theta + \frac{1}{9} \sin 9\theta + \frac{1}{27} \sin 13\theta + \dots$$

- (b) Show that $C + iS = \frac{3e^{i\theta}}{3 - e^{4i\theta}}$. [4]

- (c) Hence show that $C = \frac{9 \cos \theta - 3 \cos 3\theta}{10 - 6 \cos 4\theta}$. [3]

16 In this question you must show detailed reasoning.

The diagram shows the curve with equation $y = \frac{x+3}{\sqrt{x^2+9}}$.



The region R, shown shaded in the diagram, is bounded by the curve, the x -axis, the y -axis, and the line $x = 4$.

- (a)** Determine the area of R. Give your answer in the form $p + \ln q$ where p and q are integers to be determined. **[6]**

The region R is rotated through 2π radians about the x -axis.

- (b)** Determine the volume of the solid of revolution formed. Give your answer in the form $\pi\left(a + b \ln\left(\frac{c}{d}\right)\right)$ where a , b , c and d are integers to be determined. **[6]**

- 17** A researcher is modelling the height of a particular type of tree over its lifetime.

Data suggests that the maximum possible height of this type of tree over its lifetime is double the height of the tree 5 years after planting.

It is given that, t years after planting a seed for this type of tree, the corresponding height of the tree is h m, and that $h = 0$ when $t = 0$.

- (a) The researcher first models the height of the tree by assuming that the rate of increase of h is proportional to $(20 - h)$, with constant of proportionality 0.2.
- (i) Write down the first order differential equation for this model. [1]
 - (ii) Show that this model predicts that the maximum possible height of the tree is 20 m. [1]
 - (iii) Show by integration that $h = 20(1 - e^{-0.2t})$. [4]
 - (iv) Determine whether this model's prediction for the height of the tree 5 years after planting is consistent with the maximum possible height of the tree being 20 m. [2]
- (b) The researcher refines the model for the height of the tree using the following second order differential equation.

$$\frac{d^2h}{dt^2} + 0.3 \frac{dh}{dt} + 0.02h = 0.4$$

- (i) Determine the general solution of this second order differential equation. [4]
- (ii) Show that the refined model also predicts that the maximum possible height of the tree is 20 m. [1]

Further research determines that the initial rate of growth of this type of tree is 2.9 metres per year.

- (iii) By applying the initial conditions to find the particular solution of this differential equation, determine whether the refined model's prediction for the height of the tree 5 years after planting is consistent with the maximum possible height of the tree being 20 m. [5]

END OF QUESTION PAPER

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